

# Analytical solutions for radiation-driven winds in massive stars. II: The $\delta$ -slow regime

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**Abstract** / Accurate mass-loss rates and terminal velocities from line-driven winds are important to obtain synthetic spectra from radiative transfer calculations. From a theoretical point of view, analytical expressions for the wind parameters and velocity profile would have many advantages over numerical calculations, that solve the complex non-linear set of hydrodynamic equations. In our previous work, we obtained an analytic description for the fast wind regime. Now, we propose a new analytical expression for the line-force term and obtain a velocity profile closed-form solution for the  $\delta$ -slow regime. Using this analytical expression, we describe a methodology to obtain the mass-loss rates. Moreover, we establish a relation between our line-force term with the known stellar and m-CAK line force parameters. To this purpose, we calculate a grid of numerical hydrodynamical models and perform a multivariate multiple regression. The numerical and analytical descriptions lead to almost the same synthetic spectra.

**Keywords** / hydrodynamics — methods: analytical — stars: early-type — stars: mass-loss — stars: winds, outflows

Müller & Vink (2008, hereafter MV08) proposed a mathematical expression for the radiative line acceleration  $\hat{g}_L(\hat{r})$  as a function of stellar radius  $\hat{r}$ , but it does not fit the  $\delta$ -slow solution (Curé et al., 2011). For this reason, we modify the expression, namely:

$$\hat{g}_L(\hat{r}) = \frac{\hat{g}_0}{\hat{r}^{1+\delta_1}} \left(1 - \frac{1}{\hat{r}^{\delta_2}}\right)^\gamma \quad (1)$$

where  $\hat{g}_0$ ,  $\delta_1$ ,  $\delta_2$  and  $\gamma$  are the fitted parameters. Using the methodology of MV08, it is possible to obtain a dimensionless differential equation of motion for the radial velocity, where the solution of this equation is based on the Lambert  $W$  function. In Araya et al. (2014) a relationship between the MV08 line-force parameters and the stellar and m-CAK line-force parameters was given for fast solutions. To derive a similar relationship, now for the  $\delta$ -slow solutions, we create a grid of m-CAK hydrodynamic models in which the grid points are selected to cover the region of the  $T_{\text{eff}}-\log g$  diagram where the B- and A-type supergiants are located. Then, we develop the relationship applying a multivariate multiple regression (Rencher & Christensen, 2012). Once we know the relationship (estimated model) for the new line acceleration parameters as a function of the stellar and m-CAK line-force parameters, we can obtain the velocity profile of the  $\delta$ -slow wind solution in terms of the Lambert  $W$ -function. Because the velocity profile is not an observable quantity, we use both the analytical and numerical solutions as input of the radiative transfer code FASTWIND (Puls et al., 2005) in

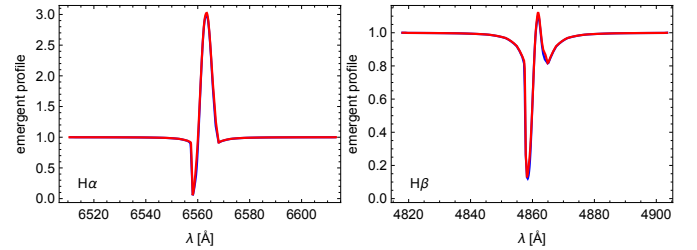


Figure 1: H $\alpha$  and H $\beta$  profiles calculated with FASTWIND. Blue and red lines correspond to the line profiles obtained using the numerical and analytical solutions, respectively.

order to obtain synthetic line profiles and compare them (Fig. 1). The results show an excellent agreement.

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